

## FE6

# ROTATION

### OBJECTIVES

#### Aims

A careful study of this chapter should equip you with a basic understanding of the concepts and principles used to describe rotational motion. You will learn how to apply these ideas to fairly simple cases where things rotate about an axis whose direction in space remains fixed. You should understand the idea of a frame of reference and the kinds of modifications to physical theory which are adopted in order to cope with situations observed from an accelerating frame of reference.

#### Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms  
*angular velocity, angular acceleration, centripetal force, translational kinetic energy, rotational kinetic energy, frame of reference, pseudoforce, centrifugal force, centrifuge, ultracentrifuge.*
2. Solve simple problems on uniform circular motion involving centripetal force, centripetal acceleration, mass, radius, speed and angular velocity.
3. Explain how the total kinetic energy of a rigid body can be expressed as the sum of translational and rotational kinetic energies and apply this result to simple problems.
4. Explain why and how the equation of motion for an accelerating frame of reference is modified by including pseudoforces
5. State and apply the formula for centrifugal force in terms of mass, angular velocity and radius.
6. Explain the operating principles of the centrifuge and ultracentrifuge.

### PRE-LECTURE

The theme of this chapter is rotation. We shall examine two quite different aspects. Firstly, extending the discussion on energy in the previous chapter, we shall look at the kinetic energy of rotating objects. Secondly, we shall look at the centrifuge. In order to talk about the centrifuge, we digress and see how the equation of motion may be written in different frames of reference; in particular, how it may be written in a rotating frame of reference.

## 6-1 CIRCULAR MOTION

Consider a small object moving with constant speed  $v$  in a circle of radius  $R$ .

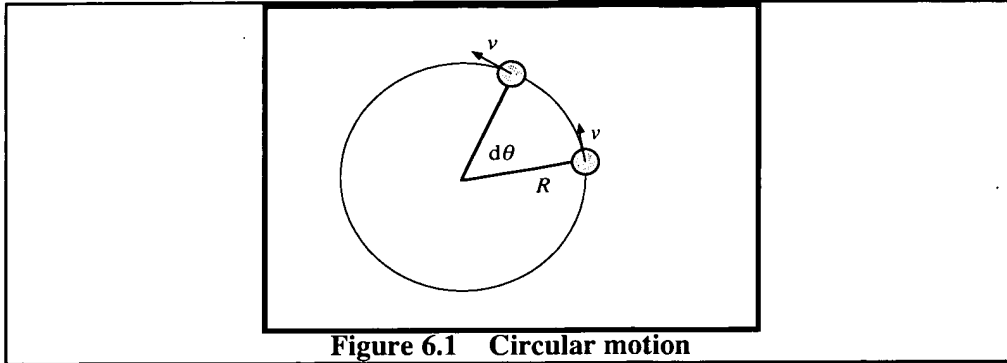


Figure 6.1 Circular motion

In a short time interval  $dt$  it sweeps out an angle  $d\theta$ . The **angular velocity** of the object is defined to be

$$\omega = \frac{d\theta}{dt} \quad \dots (6.1)$$

The angular velocity and the speed of the object are connected by the relation

$$\omega = \frac{v}{R} \quad \dots (6.2)$$

If the angular velocity changes with time the object also has an **angular acceleration**:

$$\alpha = \frac{d\omega}{dt} \quad \dots (6.3)$$

The SI unit of angular velocity is the radian per second, symbol  $\text{rad.s}^{-1}$ . The unit of angular acceleration is radian per second per second,  $\text{rad.s}^{-2}$ .

## 6-2 ROTATION OF A RIGID BODY ABOUT A FIXED AXIS

The reason for introducing these angular variables is to simplify the description of rotating bodies. If a rigid body rotates about an axis which is fixed in space, then different parts of the body have different linear velocities and accelerations. On the other hand, all the particles in the body have the same angular velocity and the same angular acceleration. This description in terms of angular variables is useful even if the axis of rotation moves.

**Q6.1** In chapter FE1 we quoted the result that the centripetal acceleration of an object moving with speed  $v$  in a circular path was equal to  $\frac{v^2}{R}$ . Re-express this in terms of angular velocity. (We need this result later.)

### Preparation

Read the sections in chapter FE3 about rotational motion and moments of inertia. Revise questions 2.4 and 2.6(b) in FE2.

## LECTURE

**6-3 ROTATIONAL KINETIC ENERGY**

When a rigid body is rotating about a fixed axis, different parts of the object move with different speeds. Parts near the edge have greater speeds than those near the axis of rotation. Consider one small part of the object, with mass  $\Delta m$ , moving at speed  $v$ . Its kinetic energy is  $\frac{1}{2}(\Delta m) v^2$ , so the total kinetic energy of the whole body is the sum of the kinetic energies of all its parts, which we can write as  $\sum \frac{1}{2}(\Delta m) v^2$ . Note that there will be many different values of  $v$  in this sum. However, by using the common angular velocity,  $\omega$ , instead of speed we can write a simple formula for the **rotational kinetic energy**:

$$\text{KE} = \frac{1}{2} I \omega^2 \quad \dots (6.4)$$

where  $I$  is the moment of inertia of the object, mentioned in chapter FE3. The moment of inertia depends on the distribution of mass in the body. If you consider two objects with the same total mass, the one with more of its mass further from the axis of rotation has the higher moment of inertia. Note also that a body does not have a unique moment of inertia; the value of  $I$  depends on the location of the axis of rotation.

In general a rigid body (e.g. a boomerang) has both rotational and translational motion. Each part still has kinetic energy  $\frac{1}{2}(\Delta m) v^2$ , but some of that energy is now associated with the motion of the body as a whole and the rest of it is associated with the rotational motion. Consequently the total kinetic energy of the body can be expressed as a sum of **translational kinetic energy**, associated with the motion of the centre of gravity of the body, and kinetic energy of rotation. Provided that the body is rigid and that its axis of rotation through the centre of gravity stays in the same direction in space, the total KE can be written in the simple form:

$$\text{total KE} = \frac{1}{2} m v_C^2 + \frac{1}{2} I \omega^2 ,$$

where  $v_C$  is the speed of the centre of gravity and  $m$  is the total mass of the body.

**Demonstrations**

- A block of dry ice slides down a slope more quickly than a round object rolls down the slope. Gravitational force does work on the objects. All the PE of the sliding object is converted into KE of translation. But the PE of the rolling object must be shared between KE of translation and KE of rotation which means that the translational motion takes only a fraction of the total KE. (The value of that fraction depends on the shape of the body, but not its size.) At a given distance down the slope, the speed of the centre of gravity must be less for the rolling object.
- A sphere rolls more quickly than a solid cylinder which in turn rolls more quickly than a hollow cylinder, even though all three objects have equal radii. This happens because the mass is distributed differently in each object and the sharing between the KE of translation and the KE of rotation is different.
- In a model car energy is stored as rotational KE of a flywheel. It is converted into translational KE of the car and eventually into non-mechanical energy which is lost from the system.

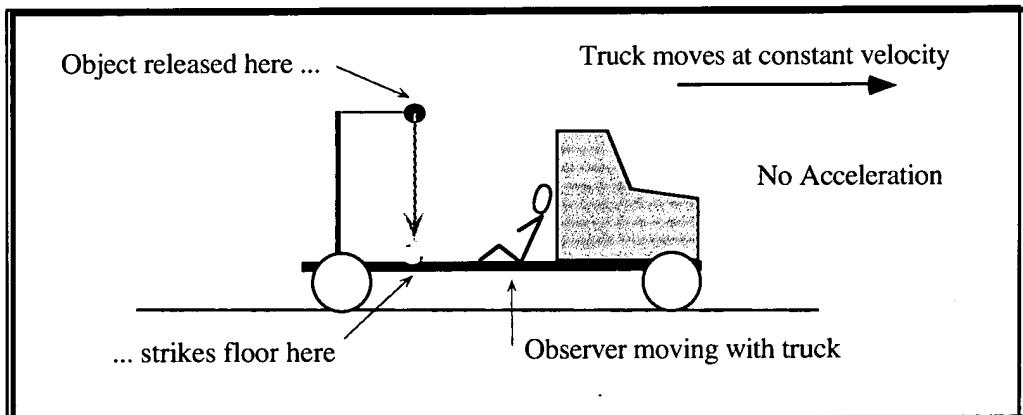
## 6-4 ACCELERATED FRAMES OF REFERENCE AND PSEUDOFORCES

### Frames of reference

If you want to describe the motion of an object you need a coordinate system, or something equivalent, to which you can relate your measurements of position; it doesn't make sense to give position coordinates ( $x, y, z$ ) unless you know where the origin is and the directions of the axes. Specifying the origin and the axes is a formal, but somewhat abstract, way of describing a **frame of reference**. In practice a frame of reference is anchored to some physical objects, and when we talk about an observer we imagine a person at rest in his or her own reference frame. We now explore some aspects of how the choice of a reference frame can affect the observed motions of objects and even the laws of motion themselves.

#### *Demonstration: Acceleration in a straight line*

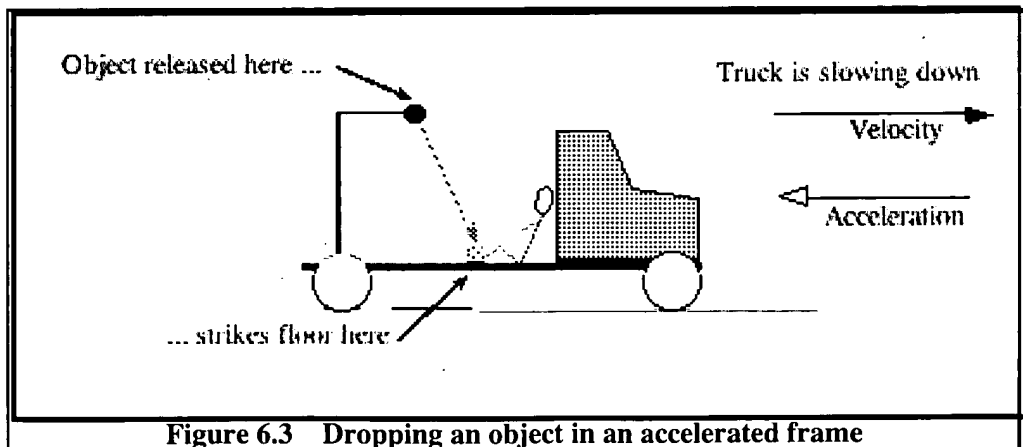
- A truck is moving at constant velocity (figure 6.2). An object is dropped and is seen to fall to a point on the floor of the truck, directly below the point of release. To an observer in the truck's frame of reference, the object falls vertically along a straight line path. The truck's frame of reference is not accelerating, and nothing unusual occurs.



**Figure 6.2 Dropping an object in a moving frame of reference**

The frame of reference attached to the truck is moving with constant velocity.

- In the second part of the demonstration the object is dropped while the truck is slowing down to stop (figure 6.3).



**Figure 6.3 Dropping an object in an accelerated frame**

In this case the truck and its frame of reference are accelerating. To an observer in the truck's frame of reference it looks as if there is a force pushing the object forward as it falls. The observer cannot explain this using the equation of motion. There is no known kind of forward force to

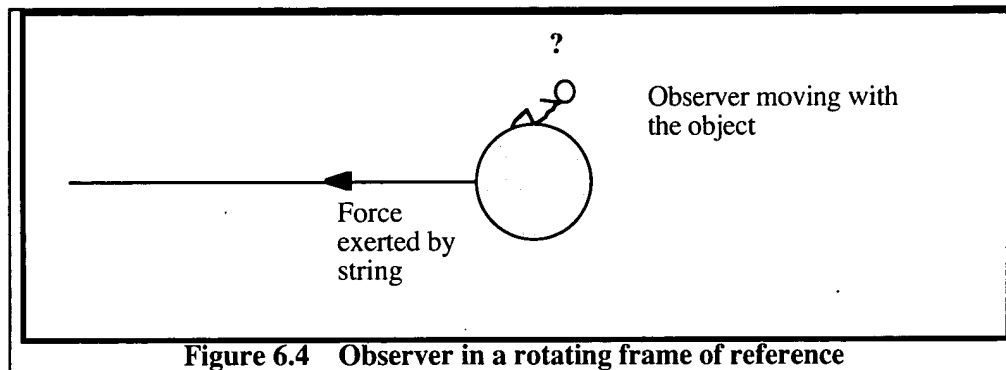
account for the path of the falling object. The problem can be resolved by introducing a **fictitious force**, or **pseudoforce**, to make the equation of motion work in the accelerated frame of reference.

**Demonstration: Measurement of weight in a lift**

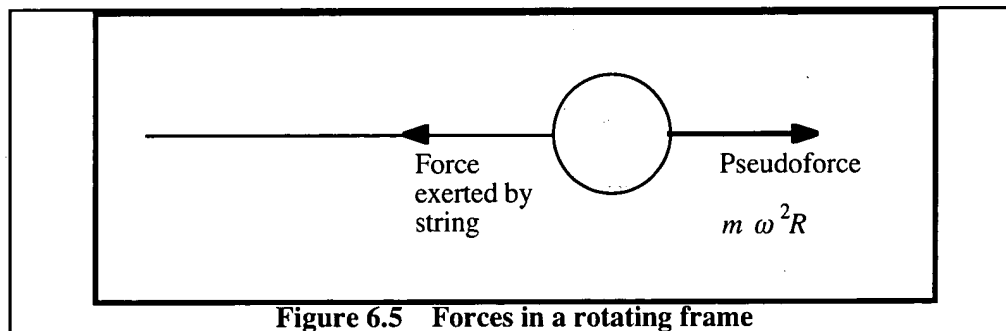
A body is weighed in a lift. (This problem was described in Q2.10, FE2, where a fixed frame of reference, outside the lift, was used.) If the frame of reference is attached to the accelerating lift, then the forces on the body being weighed are the gravitational force, the supporting force exerted by the platform of the scales, plus a pseudoforce, associated with the accelerated frame (figure 2.11). The supporting force, whose magnitude is equal to the apparent weight recorded on the scales depends on both the gravitational force and the pseudoforce.

**Motion in a circle**

Imagine an object which has a string tied to it and is swung around in an approximately horizontal circular path (figure 6.4). Think of an imaginary observer riding on the object. The frame of reference for this observer is accelerating. Now the observer knows that the string exerts a force to the left, but cannot explain why the object does not go off in that direction, although the equation of motion predicts that it should. To make the equation of motion work the observer introduces a pseudoforce - in this case a force to the right in order to counteract the force exerted by the string.



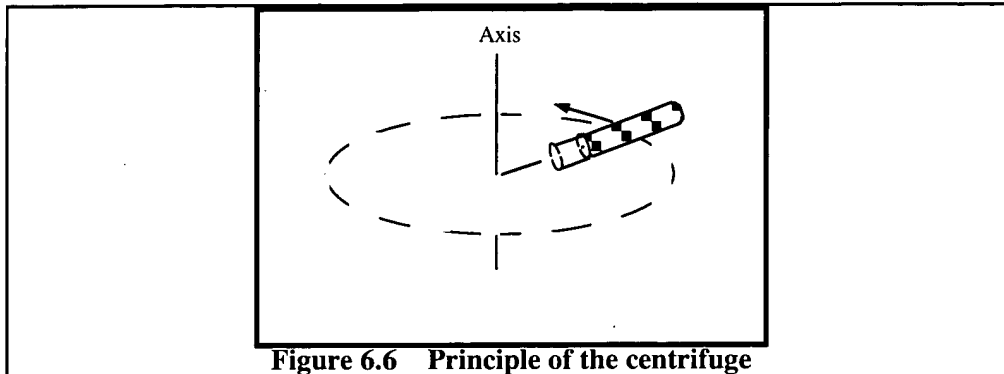
We know from our bird's eye view of the object moving in the circle that the force exerted by the string is equal to  $m\omega^2 R$ . Therefore the moving observer's pseudoforce must also be equal to  $m\omega^2 R$ , but it is in the opposite direction - outwards. This kind of pseudoforce is often called **centrifugal force**.



In summary, the equation of motion works properly only in a frame of reference that is not accelerating. However it is often convenient, from the point of view of mathematical simplicity, to use an accelerated reference frame and non-physical pseudoforces which are invented only in order to preserve the equation of motion. We say that the pseudoforces are non-physical firstly because they violate the law that forces always occur in pairs and secondly because it is not possible to identify a physical object which is the source of the force.

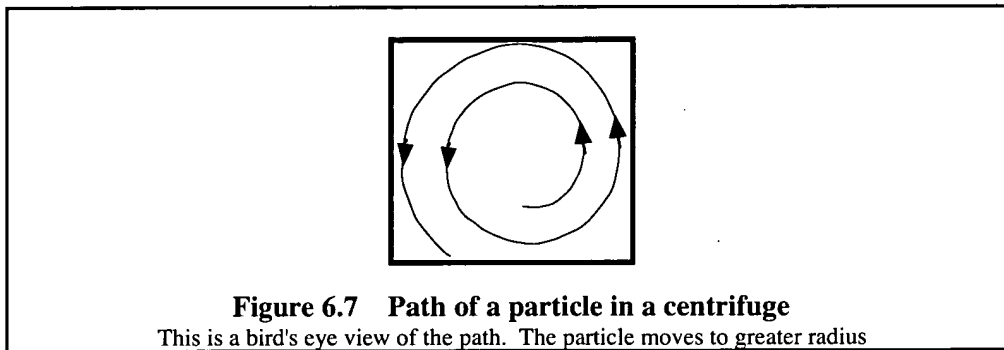
## 6-5 THE CENTRIFUGE

The **centrifuge** is used extensively to separate materials of differing densities. A tube containing the materials swings around the axis of the centrifuge at a high angular velocity (figure 6.6).



**Figure 6.6 Principle of the centrifuge**

If a suspended particle is going to remain at a given radius as the centrifuge rotates, the forces on it due to the surrounding material must combine to provide a centripetal force  $m\omega^2 R$ . If the surrounding material cannot provide sufficient force the particle moves to a greater radius where the surrounding material or the bottom of test-tube can provide the necessary centripetal force. (See figure 6.7.)

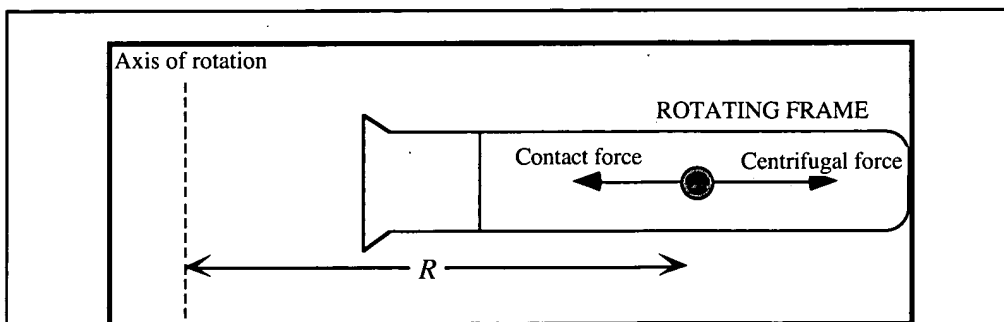


**Figure 6.7 Path of a particle in a centrifuge**

This is a bird's eye view of the path. The particle moves to greater radius

Our discussion of the centrifuge can be greatly simplified by looking at things from the point of view of an imaginary observer who rides around with the test tube rather than from a bird's eye view. An observer riding around with the tube in the centrifuge is accelerating so we can use the equation of motion ( $F = ma$ ) only by introducing a fictitious centrifugal force.

Consider the motion of a suspended particle as described in the accelerated frame of reference. In figure 6.8 the force to the left is provided by the surrounding material.



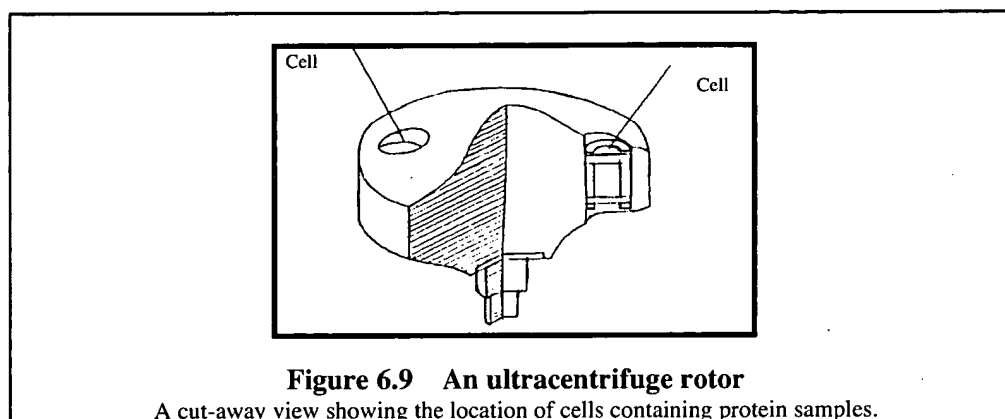
**Figure 6.8 Real force and pseudoforce on a particle in a centrifuge**

In this case the particle is suspended at a fixed radius, so it is at rest in the rotating frame. The centrifugal force is a pseudoforce. The buoyant-like contact force is a real pressure force exerted by the surrounding fluid

The pseudoforce is still equal to  $m\omega^2 R$ . If the real contact force provided by the surrounding material is less than or equal to  $m\omega^2 R$  then the particle moves further down the test-tube. The centrifugal pseudoforce is rather like a gravitational force which causes suspended particles to settle out. By making the angular velocity ( $\omega$ ) very high the pseudoforce ( $m\omega^2 R$ ) can be  $10^4$  to  $10^5$  times greater than the weight of the particle. So we can ignore the weight, giving a total force along the radius. Seen from the rotating frame, the motion of a particle that starts from rest will therefore be a straight line, not the spiral seen by the outside observer.

## 6-6 THE ULTRACENTRIFUGE

The ultracentrifuge in the Department of Biochemistry is often used to measure the molecular weights of protein molecules. The rotor shown in the lecture runs at speeds up to 60 000 revolutions per minute giving a maximum centrifugal force of 260 000 times that of gravity. Rotors for ultracentrifuges must be carefully designed so that they do not fly apart under the large centrifugal forces. These considerations set a limit on the radius of a rotor and its speed of operation. The chamber containing the rotor is evacuated to eliminate heating due to air friction.



In this case, the particles of interest are very small - small enough for diffusion to occur; so they are also small enough to be influenced by the forces exerted by individual liquid molecules rather than by the 'averaged out' forces (the drag and the buoyant forces). Sedimentation of the protein molecules cannot take place under gravity because their weight is not sufficient to overcome their diffusion upwards. If we centrifuge the sample, we can increase the speed of rotation until the centrifugal (pseudo) force overcomes the diffusion and drives the protein molecule to a greater radius.

The molecular weight of the protein molecules is determined from the sedimentation rate, the density of the molecules and the density of the surrounding fluid. Diffusion effects must be subtracted out. The size of the diffusion effects is obtained from measurements of the sedimentation rate taken at different speeds of rotation.

The measurements are all made while the sample is spinning around inside the machine. This is the difference between an analytical ultracentrifuge and the ordinary type of centrifuge which is used for separation. The radial movement of the protein molecules is monitored by optical techniques. Use is made of the fact that the refractive index of the protein molecules is different from that of the surrounding liquid.

## 6-7 CORIOLIS FORCE

Another kind of pseudoforce, the Coriolis force, appears in a rotating frame of reference. A demonstration shows a two-dimensional version of the Coriolis effect. A container moving along a straight path drops sand onto a rotating platform. The track of the falling sand in the frame of the

platform is a curve rather than a straight line. To explain this curved path another fictitious force, the Coriolis force, can be introduced.

A similar effect occurs with global wind patterns. Air masses moving towards the equator are deflected from a true south-north path because the Earth rotates underneath. In a rotating frame attached to the Earth's surface, the winds appear to be deflected. In order to preserve the equation of motion in the rotating frame we invent two fictitious forces - centrifugal and Coriolis.

## POST-LECTURE

### 6-8 QUESTIONS

#### Rotation

- Q6.2** A gramophone record is rotating at a rate of  $33\frac{1}{3}$  revolutions per minute. What is its angular velocity in  $\text{rad.s}^{-1}$ ?
- Q6.3** The dental drill probably reaches higher speed than any other production micro-mechanism. The angular speed is about  $60\,000\text{ rad.s}^{-1}$ . If the dental burr has a diameter of 1.5 mm, how fast is the outside of the burr moving?

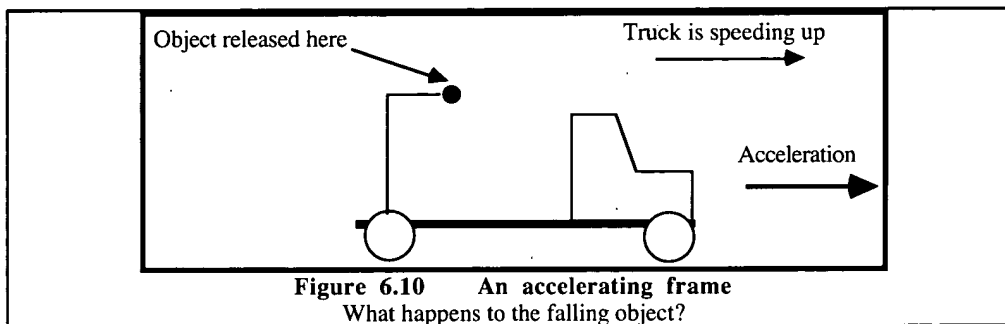
#### Rotational kinetic energy

- Q6.4** A flywheel is rotating with an angular velocity of 5 revolutions per second. What does its angular velocity become when its rotational kinetic energy increases 16 times?
- Q6.5** Could a sphere roll down a slope if friction were absent?
- Q6.6** How can you tell the difference between a raw egg and a hard-boiled egg by spinning them?

#### Accelerated frames of reference and pseudoforces

- Q6.7** Refer to the demonstrations of motion observed in a frame of reference attached to a moving truck.

- a) If the truck were speeding up (figure 6.10) where would the object strike the floor?



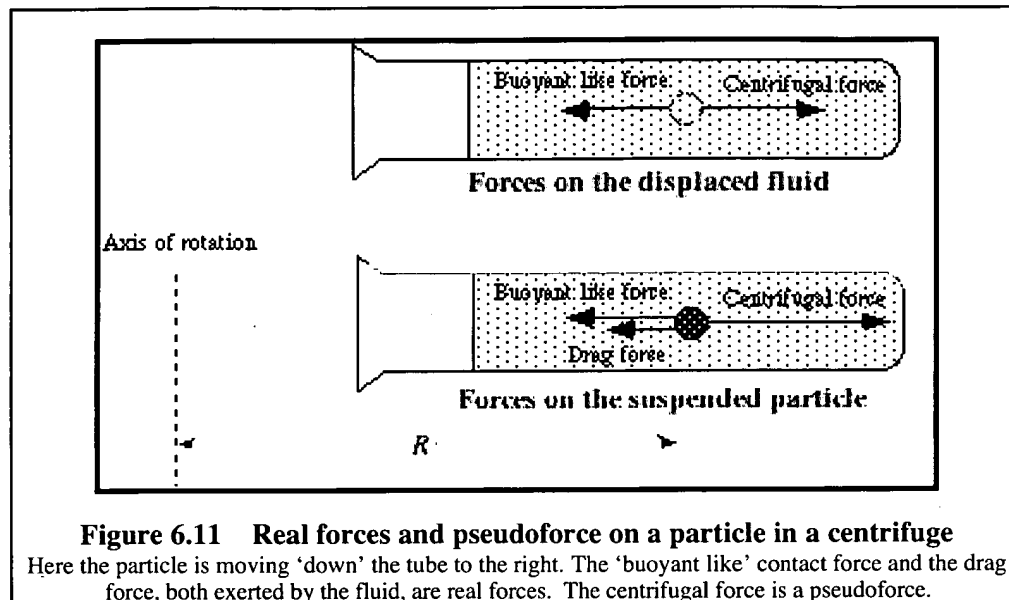
- b) Can an observer standing at the side of the road explain the motion using the equation of motion? (Or does the observer need to invoke pseudoforces?)
- Q6.8** Discuss the way you are thrown around in a car when it stops suddenly and when it turns a corner sharply
- from the point of view of a bird watching from its perch on a power line, and
  - from the point of view where all measurements are made with respect to the car.

### 6-9 MORE ABOUT THE CENTRIFUGE

The centrifuge is used to increase the settling speed of particles which fall very slowly through the surrounding liquid. (See §4-5 on *Sedimentation*). In a frame of reference rotating with the test-tube the horizontal forces on a typical particle are as shown in the horizontal plan view in figure 6-11. In comparison with these forces, the vertical forces on the particle are very small and may be neglected. (The Coriolis force can also be neglected in this argument.) The two forces acting to the left are exerted by the surrounding liquid. The drag force is the same as that introduced in FE4. The



'buoyant-like' force is the analogue of the familiar buoyant force - its magnitude is independent of the motion of the particle relative to the liquid. Both are real forces.



**Figure 6.11 Real forces and pseudoforce on a particle in a centrifuge**

Here the particle is moving 'down' the tube to the right. The 'buoyant like' contact force and the drag force, both exerted by the fluid, are real forces. The centrifugal force is a pseudoforce.

The magnitude of the buoyant-like force can be found by the method used for the buoyant force in FE3. Consider the forces which would act on a portion of fluid, mass  $m_L$ , corresponding to the liquid displaced by the particle. See the upper part of figure 6.11. Since this portion of liquid does not move along the radius (why?), the 'buoyant like' force balances the centrifugal force  $m_L \omega^2 R$ . Using  $\rho_L$  to denote the density of the liquid and  $V$  for the common volume of the particle and the displaced liquid we get:

$$\text{buoyant-like force} = m_L \omega^2 R = \rho_L V \omega^2 R.$$

Returning to the forces on the suspended solid particle (lower diagram in figure 6.11), the buoyant like force on the particle is equal to that on the displaced liquid. (Why?) Also

$$\text{centrifugal force on the particle} = \rho V \omega^2 R,$$

where  $\rho$  is the density of the particle.

The terminal velocity is the velocity at which the drag force  $\lambda v$  balances the other forces acting. So the equation of motion for the suspended particle becomes:

$$\rho_L V \omega^2 R + \lambda v_T = \rho V \omega^2 R.$$

We can rearrange this to get an expression for the settling speed:

$$v = \frac{V}{\lambda} (\rho - \rho_L) \omega^2 R.$$

Compare this with the equation for gravitational settling worked out in §4-5 :

$$v_T = \frac{V}{\lambda} (\rho - \rho_L) g.$$

The angular speed  $\omega$  of the centrifuge can be increased to make the centripetal acceleration,  $\omega^2 R$ , very much greater than  $g$  and to provide very much greater settling speeds.

**Q6.9** A general laboratory centrifuge rotates at its maximum angular speed of 8 000 revolutions per minute. Consider a particle at a rotation radius of 110 mm.

- Compare the centrifugal force on the particle with its weight.
- In Q4.1, FE4, the sedimentation rate for a particle settling under gravity was several millimetres per hour. What would the sedimentation rate be if this centrifuge were used?

<b>SUMMARY: GRAPHICAL PRESENTATION OF INFORMATION</b>
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The table below shows how slopes and areas of various kinds of graphs can be used to calculate values of physical quantities.

Ordinate ('y')	Abscissa ('x')	Slope	Area between curve and 'x' axis
distance travelled	time	speed	-----
displacement*	time	velocity*	-----
speed	time	-----	distance travelled
velocity*	time	acceleration*	displacement*
acceleration*	time	-----	change in velocity* (v)
total force*	time	-----	mass $\times$ (change in velocity*)
force*	displacement*	-----	work
conservative force*	displacement*	-----	- (change in potential energy)
potential energy	displacement*	-(conservative force*)	-----